

Midterm Exam # 1 - 60 Points

The exam is closed book and closed notes. Please show your work step by step. Simple calculators may be used (no graphing calculators and no smart phones or iPods)

You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature _____

Problem 1 (10 points)

- a.) Tim Lincecum has pitched three games, giving up 2, 4, and 3 earned runs in each. The first two games were at home, and the last away. In each, he pitched 7, 6, and 5 innings. Please calculate the correlation between earned runs and innings pitched. (10 Points)

$$\begin{aligned} \hat{\mu}_R &= \frac{1}{3}(2+4+3) = 3 & \hat{\sigma}_R &= \sqrt{\frac{1}{2}((2-3)^2 + (3-3)^2 + (4-3)^2)} & +1 \\ & & &= \sqrt{\frac{1}{2}(1+0+1)} = 1 & \\ \hat{\mu}_I &= \frac{1}{3}(7+6+5) = 6 & \hat{\sigma}_I &= \sqrt{\frac{1}{2}((7-6)^2 + (6-6)^2 + (5-6)^2)} & \\ & & &= \sqrt{\frac{1}{2}(1+0+1)} = 1 & +1 \end{aligned}$$

$$\begin{aligned} \hat{\sigma}_{RI} &= \frac{1}{2}((2-3)(7-6) + (4-3)(6-6) + (3-3)(5-6)) & +2 \\ &= \frac{1}{2}(-1 \cdot 1 + 1 \cdot 0 + 0 \cdot (-1)) \\ &= -0.5 \end{aligned}$$

+2 for work

$$\hat{\rho}_{RI} = \frac{\hat{\sigma}_{RI}}{\hat{\sigma}_R \cdot \hat{\sigma}_I} = \frac{-0.5}{1 \cdot 1} = \boxed{-0.5}$$

b.) Suppose that I measure pitching duration in games rather than innings. Hence, he pitches 7/9, 6/9, and 5/9 innings. What is the new correlation and why? (10 points)

$$\hat{\rho}_{R_{G}} = \hat{\rho}_{R_{I}} = -0.5$$

correlation is insensitive to scale

or work from previous problem with same point breakdown

Problem 2 (20 Points)

Suppose that study hours per week follow a uniform distribution between 0 (!!!) and 35.

- a. What is the probability that a randomly selected student studies 10 hours per week? (5 points)

0

- b. Suppose that event A is when a student studies 10 hours or more per week and event B is that a student studies 20 hours or more per week. Are these events independent? (5 points)

No +5

Knowing that a student studies less than 10 hours per week changes the probability that he/she studied 20 hours or more per week

- c. Please calculate $\Pr(A|B)$. (10 points)

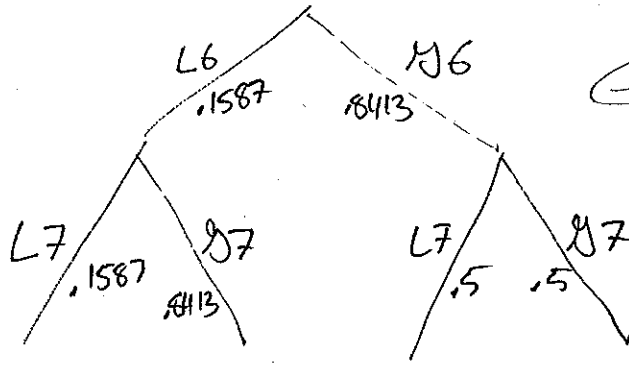
$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\int_{20}^{35} \frac{1}{35} dx}{\int_{20}^{35} \frac{1}{35} dx}$$
$$= \frac{15/35}{15/35} = \boxed{1}$$

Problem 3 (20 Points)

a. Suppose that hours-of-sleep follows a normal distribution with mean 7 and standard deviation 1. What is the probability on a given night of sleep a person gets between 6 and 9 hours of sleep? (10 points)

$$\begin{aligned} & P_r(6 < \text{Sleep} < 9) \\ & = P_r(-1 < Z < 2) \quad +11 \end{aligned} \quad \left. \begin{array}{l} z_6 = \frac{6-7}{1} = -1 \\ z_9 = \frac{9-7}{1} = 2 \end{array} \right\}$$
$$\begin{aligned} & = P_r(Z < 2) - P_r(Z < -1) \\ & = P_r(Z < 2) - P_r(Z > 1) \\ & = P_r(Z < 2) - (1 - P_r(Z < 1)) \quad +6 \\ & = 0.9772 - (1 - 0.8413) \\ & = \boxed{0.8185} \end{aligned}$$

- b. While we normally assume that things are independent over time, they are usually not in practice. In this case, suppose that the first night of sleep is characterized by the distribution in part (a). If a person gets greater than 6 hours of sleep in the first night, the second night is characterized by the same distribution. If a person gets less than 6 hours of sleep in the first night, the second night is characterized by a normal distribution with mean 8 and standard deviation 1. Given that this person got 7 hours of sleep or more in the second night, what is the probability that he/she got more than 6 hours of sleep in the first night? (10 points)



$$Z_6 = \frac{6-7}{1} = -1$$

← Same as previous question

$$Z_7 = \frac{7-7}{1} = 0$$

$$\Rightarrow \Pr(S < Z) = 0.5$$

$$Z_7 = \frac{7-8}{1} = -1$$

same as (6a)

$$\Pr(Y_6 | Y_7) = \frac{\Pr(Y_6 \cap Y_7)}{\Pr(Y_7)}$$

$$= \frac{0.8413 \cdot 0.5}{0.8413 \cdot 0.5 + 0.1587 \cdot 0.8413}$$

$$\Pr(Y_6 | Y_7) = 0.759$$